Biol. Cybern. 64, 353–356 (1991)

Biological Cybernetics

An artificial hysteresis binary neuron: a model suppressing the oscillatory behaviors of neural dynamics

Y. Takefuji and K. C. Lee

Department of Electrical Engineering and Applied Physics, Case Western Reserve University, Cleveland, OH 44106, USA

Received April 24, 1990/Accepted in revised form November 7, 1990

Abstract. A hysteresis binary McCulloch–Pitts neuron model is proposed in order to suppress the complicated oscillatory behaviors of neural dynamics. The artificial hysteresis binary neural network is used for scheduling time-multiplex crossbar switches in order to demonstrate the effects of hysteresis. Time-multiplex crossbar switching systems must control traffic on demand such that packet blocking probability and packet waiting time are minimized. The system using $n \times n$ processing elements solves an $n \times n$ crossbar-control problem with $O(1)$ time, while the best existing parallel algorithm requires $O(n)$ time. The hysteresis binary neural network maximizes the throughput of packets through a crossbar switch. The solution quality of our system does not degrade with the problem size.

Introduction

In time-multiplex communication systems, crossbar packet switches route traffic from the input to output where a message packet is transmitted from the source to the destination. The randomly incoming traffic must be controlled and scheduled to eliminate conflict at the crossbar switch where the conflict is that two or more users may simultaneously access to a single output. The goal of the traffic-scheduling for time-multiplex crossbar switches is not only to maximize the throughput of packets through a crossbar switch but also to minimize packet blocking probability and packet waiting time.

A request for packet transmission through an $n \times n$ crossbar is described by an $n \times n$ traffic matrix T. In the traffic matrix T, each element t_{ij} represents a request of packets from input i to output j. For example, $t_{ij} = 0$ means that there is no packet to be transmitted on the jth output line from the ith input line. $t_{ij} = 1$ means that at least one packet on the ith input line should be transmitted on the jth output line of the crossbar.

In 1979 Inukai at COMSAT lab. proposed the $O(n^2)$ sequential algorithm for the $n \times n$ crossbar switch problem (Inukai 1979). In 1989 Rose at AT&T Bell lab. presented the $O(n)$ parallel algorithm based on a cellular automaton where n^2 processing elements are used for solving an $n \times n$ traffic matrix problem (Rose 1989). Chen, Mavor, Denyer, and Renshaw proposed the $O(n^2)$ sequential algorithm of traffic routing problems for the multiprocessor system (Chen et al. 1990). They proved that the problem is NP-complete (Chen et al. 1990). In 1989 Marrakchi and Troudet at Bellcore proposed the $n \times n$ neural network algorithm based on Hopfield network model (Marrakchi and Troudet 1989). However with the Hopfield neural network, the state of the system is forced to converge to the local minimum. In other words, the solution quality drastically degrades with the problem size. Takefuji and Lee have successfully used the Hopfield neural network with McCulloch–Pitts binary neurons for solving the graph planarization problem (Takefuji and Lee 1989) and the tiling problem (Takefuji and Lee 1990a) where the state of the system converges to the near-global minimum in $O(1)$ time. They proved that the state of the binary neural network system is guaranteed to converge to the local minimum (Takefuji and Lee 1990b). In 1986 Hoffman and Benson introduced sigmoid neurons with hysteresis for learning, where any changes in synaptic connection strengths are replaced by hysteresis (Hoffman and Benson 1986). Due to the hysteresis associated with each neuron, the system tends to stay in the region of phase space where it is located. They proposed the theory on a role for sleep in learning (Hoffman and Benson 1986).

Dynamic and static hysteresis in Crayfish stretch receptors was reported by Segundo and Martinez in 1985 (Segundo and Martinez 1985). They stated that hysteresis may be more widespread than suspected in sensory and perhaps other system. In 1989 Keeler, Pichler, and Ross presented the effects of hysteresis in pattern recognition and learning for improving the signal-to-noise ratio (Keeler et al. 1989). In this paper the hysteresis property is exploited in order to reduce the complicated oscillatory behaviors of neural dynamics for solving combinatorial optimization problems. The hysteresis with each neuron enhances the state of the system to stay in the region of phase space where it is located. In other words, it suppresses the oscillatory behaviors of neural dynamics so that the convergence time to the global minimum is drastically shortened.

The McCulloch–Pitts neuron model is a binary unit whose value depends on the linear sum of weighted inputs from the other neurons in the network (McCulloch and Pitts 1943). Figure 1a shows the input/output relation of the McCulloch–Pitts neuron model. In 1982 Hopfield proposed the continuous input/output unit called the sigmoid neuron model (Hopfield 1982) as shown in Fig. 1b. Simic presented the molecular electronic device with hysteresis in 1986 where it has the sigmoid hysteresis (Simic 1986).

In this paper a binary neuron model with hysteresis is introduced. Figure 1c shows the input/output relation of the hysteresis binary neuron model. A binary neural network with hysteresis is used for scheduling time-multiplex crossbar switches in order to demonstrate the effects of hysteresis. The complicated oscillatory behavior is one of the most undesirable phenomena in neural dynamics for solving optimization problems where we lack the mathematical tools to manipulate and understand them at a computational level (Hopfield and Tank 1986). Hysteresis suppresses the oscillatory behaviors of neural dynamics and consequently it shortens the convergence time to the global minimum. In other words, hysteresis in individual neurons allows the state of the proposed neural network to converge to the global minimum in $O(1)$ time. The system uses an $n \times n$ neural network array for solving an $n \times n$ crossbar switch problem where the output of the ijth hysteresis neuron V_{ij} is given by: $V_{ij} = 1$ if $U_{ij} > \text{UTP}$ (upper trip point), 0 if $U_{ij} < \text{LTP}$ (lower trip point). Note that U_{ij} is the input of the ijth neuron. The output at any particular time does not depend only upon the present value of the input but also upon past values.

The proposed hysteresis binary neural network not only maximizes the throughput of packets through a crossbar switch but also minimizes packet blocking probability and packet waiting time. The constraints on an $n \times n$ crossbar switch are that no two inputs may be connected to the same output simultaneously and that no one input may be connected to more than one output simultaneously. In other words, no two packets should share the same row and the column of the traffic matrix.

The system simulator was developed based on the proposed model. A large number of simulation results are shown and demonstrated in order to support the effects of hysteresis.

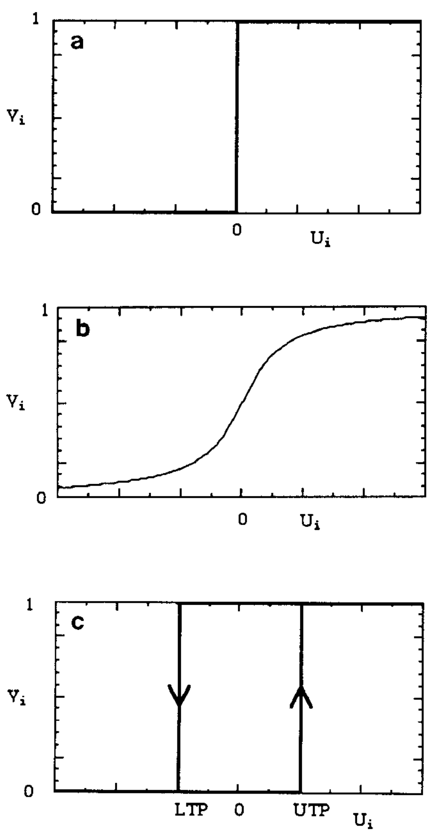

Fig. 1. **a** Binary, **b** sigmoid, and **c** hysteresis binary functions

Neural network representation

The constraints are considered that no two packets should share the same row and the column of the $n \times n$ traffic matrix. Our system uses an $n \times n$ neural network array where the motion equation of the ijth neuron is given by:

$$\frac{dU_{ij}}{dt} = -A\left(\sum_{k=1}^{N} V_{ik} - 1\right) - A\left(\sum_{k=1}^{N} V_{kj} - 1\right) + Bh\left(\sum_{k=1}^{N} V_{ik}\right) + Bh\left(\sum_{k=1}^{N} V_{kj}\right) \quad (1)$$

where $h(x)$ is called the hill-climbing term, $h(x)$ is 1 if $x = 0$, 0 otherwise. Note that coefficients A and B are constant. The first term and the second term are the row constraint and the column constraint respectively. The first term forces one and only one neuron to be fired per row. If no neuron is fired per row or per column then it will perform excitatory forces. If more than one neuron are fired per row or per column then it will act inhibitory forces. The third term and the last term are the row hill-climbing term and the column hill-climbing term respectively. The hill-climbing terms are activated only when local conflicts (where no neuron is fired per row or column) are detected. If there is no conflict the hill-climbing terms will perform no operation. In other words, the local conflicts are resolved by the hill climbing terms where the ijth neuron is encouraged to fire if the neuron has conflicts.

Parallel algorithm of the hysteresis neural network

The following procedure describes the proposed parallel algorithm.

0. Set $t = 0$.

1. The state of the input $U_{ij}(t)$ for $i = 1, \ldots, N$ and $j = 1, \ldots, N$ is randomized and evaluate the output $V_{ij}(t)$ for $i = 1, \ldots, N$ and $j = 1, \ldots, N$: $V_{ij}(t) = 1$ if $U_{ij}(t) > 0$, 0 otherwise. Goto step 3.

2. Evaluate the output $V_{ij}(t)$ for $i = 1, \ldots, N$ and $j = 1, \ldots, N$:

$$V_{ij}(t) = 1 \text{ if } U_{ij}(t) > \text{UTP}, \ 0 \text{ if } U_{ij}(t) < \text{LTP}.$$

3. Compute $U_{ij}(t+1)$ for $i = 1, \ldots, N$ and $j = 1, \ldots, N$ based on the firsr order Euler method:

$$U_{ij}(t+1) = U_{ij}(t) + DU_{ij}(t)$$

where

$$DU_{ij}(t) = -A\left(\sum_{k=1}^{N} V_{ik}(t) - 1\right) - A\left(\sum_{k=1}^{N} V_{kj}(t) - 1\right) + Bh\left(\sum_{k=1}^{N} V_{ik}(t)\right) + Bh\left(\sum_{k=1}^{N} V_{kj}(t)\right)$$

4. If $U_{ij}(t+1) > U\_\max$ then $U_{ij}(t+1) = U\_\max$
If $U_{ij}(t+1) < U\_\min$ then $U_{ij}(t+1) = U\_\min$

5. If all the conflicts are resolved then terminate this procedure else increment t by 1 and go to step 2.

The simulator has been developed on a Macintosh SE/30 and a DEC3100 workstation. Figure 2a shows the relation between the number of iteration steps, the problem size, and the band size of hysteresis. The hysteresis band size is given by the hysteresis band size $= |\text{UTP}| = |\text{LTP}|$. When no hysteresis is given to each neuron, it usually takes more than 5000 iteration steps or does not converge to the global minimum at all. Figure 2b zooms up the part of Fig. 2a where a range of the hysteresis band size is one through five. Figure 2a and b shows that the state of the system can converge to the global minimum in $O(1)$ time. The hysteresis band size = 1, 2 or 3 gives the relatively good convergence to the global minimum.

Figure 3 shows the relation between the average number of iteration steps and the problem size where 10×10 crossbar problems through 150×150 crossbar problems were investigated. Note that 100 simulation runs were performed to obtain the one average number of iteration steps per problem. The simulation result depicts that the state of the system converged to the solution within several hundred iteration steps. The solution quality of a hysteresis binary neural network with small hysteresis does not degrade with the problem size as far as we have observed. Figure 4a–d show several solutions for $n \times n$ crossbar problems. The following parameters were used in our simulation: $A = 1$, $B = 1$ if $(t \bmod 10) < 2$, 0 otherwise, $U\_\min = -800$, and $U\_\max = 200$. Based on our simulation results the hysteresis in individual neurons effectively suppresses the oscillatory behaviors of neural dynamics. Consequently the convergence time to the solution is shortened.

The oscillatory behavior can be observed when more than one neuron simultaneously and alternatively fire on and off. We believe that the hysteresis in individual neurons reduces chances for neurons to simultaneously firing on or off. Suppose that the state of oscillatory neurons is in the hysteresis band. The other neurons have chances to compete with them and their

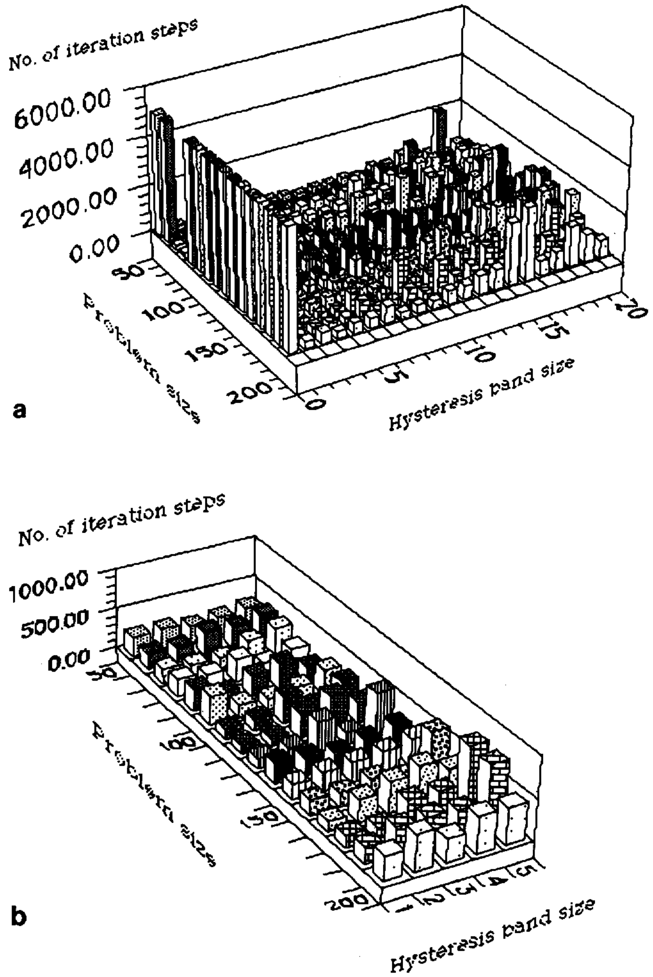

Fig. 2. **a** The relation between the number of iteration steps, the problem size, and the hysteresis band size. **b** The relation between the number of iteration steps, the problem size, and the hysteresis band size

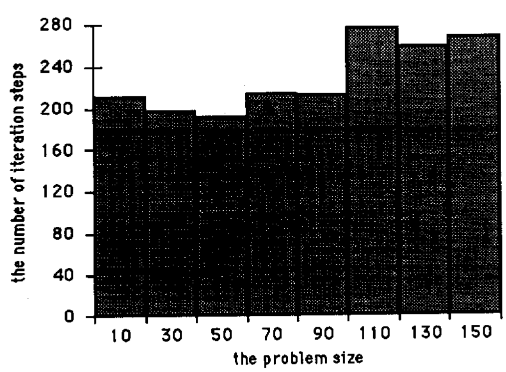

Fig. 3. The relation between the average number of iteration steps and the problem size when the hysteresis band size is 3

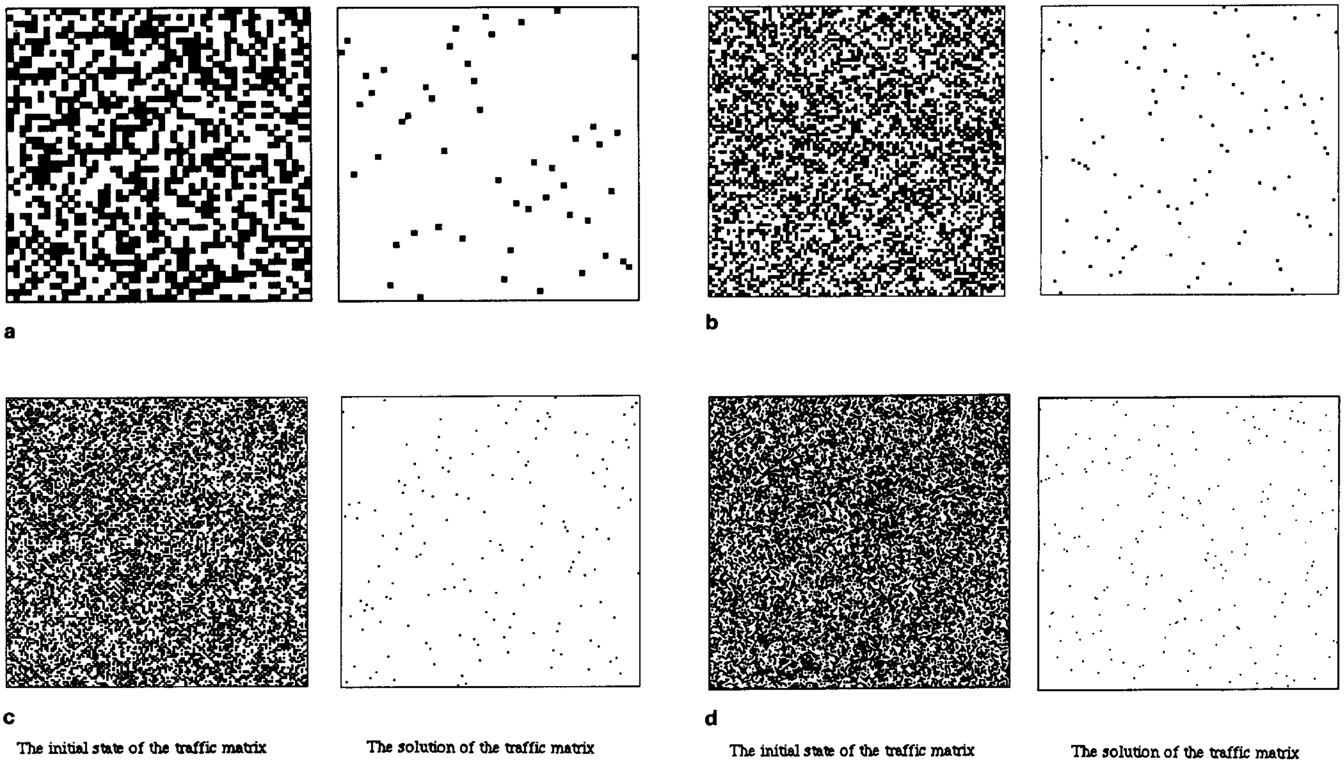

Fig. 4. a The initial state of a 50 × 50 traffic matrix and the solution. The black square represents nonzero in the traffic matrix. **b** The initial state of a 100 × 100 traffic matrix and the solution. The black square represents nonzero in the traffic matrix. **c** The initial state of a 150 × 150 traffic matrix and the solution. The black square represents nonzero in the traffic matrix. **d** The initial state of a 200 × 200 traffic matrix and the solution. The black square represents nonzero in the traffic matrix

competition disturbs the oscillatory behavior. If the oscillation is disturbed or alleviated, the convergence time to the solution can be shortened.

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Prof. Yoshiyasu Takefuji
Case Western Reserve University
Case Institute of Technology
Dept. of Electrical Engineering and Applied Physics
Glennan Building
Cleveland, OH 44106
USA